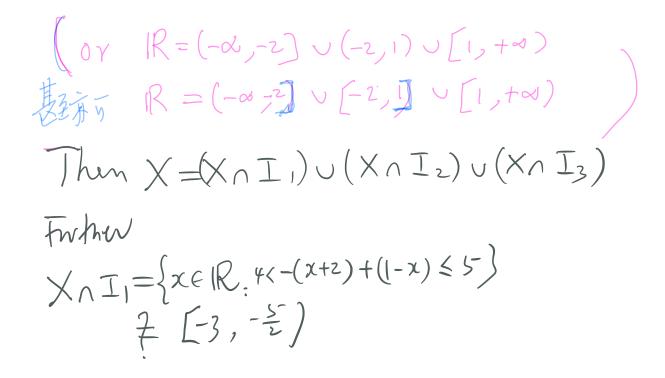
3. Without assuming III, Show that  $- \sup X = \inf (-X)$ provided that either sup X exists in IR or mig (-X) exists in IR. 4. Let  $\emptyset \neq A$ ,  $B \subseteq \mathbb{R}$  and  $A+B = \{a+b: a\in A, b\in B\}$ Show mat sup(A+B) = sup A + sup B provided hat sup A and sup B exist in IR.

6. Let 
$$a, b, x_1 > o$$
 (each positive), and  
 $x_{n+1} = x_n + \frac{1}{x_n} \quad \forall n \in \mathbb{N}.$   
Show:  
(i)  $a^2 < b^2 \quad iff (= if and only if) \quad a < b.$   
(ii)  $x_{n+1} > x_n \quad and \quad x_{n+1} > x_n^2, \quad \forall n \in \mathbb{N}.$   
(iii)  $x_{n+1} > x_n \quad and \quad x_{n+1} > x_n^2, \quad \forall n \in \mathbb{N}.$   
(iii)  $x_{n+1}^2 > x_{n+1} \cdot x_n = x_n^2 + 1 \quad and \quad m_{n+1}^2 = n, \quad \forall n \in \mathbb{N}.$   
(iv)  $Aequences(x_n^2) \quad and(x_n) \quad ave not bounded.$   
(iv)  $Aequences(x_n^2) \quad and(x_n) \quad ave not bounded.$   
(iv)  $f(x_n) = \binom{n-1}{k} + \binom{n-1}{k-1} \quad (k \leq n, natural nos.),$   
Show the Binomial Theorem and Bernoully: Inequality for  
 $a, b > o \quad (\forall k, n \in \mathbb{N} \text{ with } k \leq n) :$   
(i)  $(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}.$   
(ii)  $(1+a)^n \ge \frac{n(n-1)\cdots(n-k+1)}{k!} a^k.$ 

8<sup>\*</sup>. "Solve" the megnality system,  
(#) 
$$4 < |x+2| + |x-1| \leq 5$$
,  
that is, let X consist of all x satisfying  
the above inequalities, conventily express X.  
Hint: Try to remove the absolute value signs  
Supervalely in each of the following cases:  
(i) both (x+2) and (x-1) are  $30$ ,  
(ii) both (x+2) and (x-1) are  $30$ ,  
(iii)  $x+2 \ge 0$  but (x-1)  $\le 0$ ,  
(iv) ....  
Let X<sub>1</sub> consist of all x satisfying (i) and (#).  
Show X<sub>1</sub> =  $\left[\frac{3}{2}, 2\right]$ . Similarly  $X_1 = \emptyset$ ,  $X_3 = \emptyset$   
and  $X_2 = \left[-3, -\frac{5}{2}\right]$ .  
Note. Your may also divide into 3 cases.  
e.g.  
 $R = (-\infty, -2) \cup \left[-2, 1\right] \cup (1, +\infty)$   
 $= I_1 \cup I_2 \cup I_3$  where  
 $I_1 := \left[-\infty, -2\right]$   
 $I_2 := \left[-2, 1\right]$   
 $I_3 := (1, \infty)$ 



$$X_{n}I_{2} = \{x \in \mathbb{R} : 4 < (x+2) + (i-x) \le 5\}$$

$$\overrightarrow{F} O$$

$$X_{n}I_{3} = \{x \in \mathbb{R} : 4 < (x+2) + (x-1) \le 5\}$$

$$\overrightarrow{F} (\frac{3}{2}, 2)$$

$$\overrightarrow{F} (\frac{3}{2}, 2)$$

$$\overrightarrow{F} (\frac{3}{2}, 2) \cup (\frac{3}{2}, 2)$$